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A heat-insulated permeable wall with suction in a compressible gas flow

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ABSTRACT

The differential model of turbulence, supplemented with the transport equation for turbulent heat flux, is used to perform a numerical investigation of the boundary layer on a heat-insulated wall with suction in a compressible gas flow. It is shown that the laminarization of the initially turbulent boundary layer occurs under conditions of suction of gas, as is evidenced both by the behavior of integral and local characteristics of the flow and heat transfer and by the degeneracy of turbulence when the suction becomes asymptotic. In so doing, the temperature recovery factor is independent of Prandtl number and becomes equal to unity, i.e., the temperature of the heat-insulated wall becomes equal to the stagnation temperature of the outer flow.

1. Introduction

The initial investigations of turbulent boundary layer by numerical methods [1] revealed the efficiency of such approaches, especially, in the case of complex boundary conditions. The use of permeable surfaces in various devices is caused by the need for heat shielding of the walls (by means of injection) or for control of the boundary layer (by means of suction). One can judge the urgency of the problem of heat shielding of the walls by the large number of publications on the investigation of boundary layer with injection (a fairly complete list of references to such publications is found in [2]). Many fewer studies are available on the suction of boundary layer than those on injection. This is apparently explained both by the complexity of setting up an experiment (especially in the case of compressible boundary layer) and by the limitations of computational models, which are incapable of correctly describing rather complex processes occurring in the case of suction, in particular, the laminarization of boundary layer.

The influence of suction on an incompressible turbulent boundary layer was studied earlier both numerically and experimentally [5]; in particular, from a flow of incompressible fluid at constant velocity u_1 under conditions of uniform suction with intensity $F = v_w/u_1$.

The three-parameter model of turbulence [3] was used in [4] for performing calculations in the range of variation of suction parameter $F = 0$–0.01. This range corresponds to the range of investigation in [5], with whose results the comparison was made in [4].

The calculations showed that the shear stress τ and the energy of turbulence E in the presence of suction decreased over the turbulent layer thickness in comparison to the case of $F = 0$. It was found both in calculations [3] and in experiments [5] that, in the case of suction, the deformation of the velocity profile was nonmonotonic: for low values of the suction parameter ($F < 0.005$), the velocity profiles became more concave than in the case of impermeable plate; with further increase in the suction parameter ($F \approx 0.01$), the velocity profiles become more extended. This is an indication (obtained for the first time in calculations) of laminarization of the boundary layer, which is further evidenced by the significant decrease in the values of shear stress τ and of energy of turbulence E. In so doing, the correlation between the friction coefficient c_f and the suction parameter has the form $c_f = 2F$, which corresponds to the laminar boundary layer with asymptotic suction [6].

No experimental data are available for compressible turbulent boundary layer with suction. As to numerical investigations, we are aware of only one study [7], where the algebraic model of turbulence is used and rather limited information is obtained on the recovery factor for a heat-insulated permeable plate with suction of gas in the range of Prandtl number values $Pr = 0.2$–0.7.

It is the objective of this study to perform a numerical investigation of a compressible boundary layer with suction on a heat-insulated permeable plate in a wide range of values of suction parameter and Prandtl number using the differential model of turbulence [4,8,9].

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Nomenclature

$b_m = j_w/\mathrm{St}$ permeability parameter
c_f friction coefficient
c_p specific isobaric heat capacity
E energy of turbulence
$e = \sqrt{E}/u_1$ intensity of turbulence
h enthalpy
$j_w = (\rho v)_w/(\rho u)_1$ intensity of suction
L integral scale of turbulence
M Mach number
Pr Prandtl number
q heat flux
Re Reynolds number
r_T recovery factor for temperature
r_h recovery factor for enthalpy
St Stanton number
T temperature
u, v projections of velocity vector on coordinates x and y

Greek symbols
δ boundary layer thickness
η dynamic viscosity
ρ density
τ shear stress

Subscripts
1 parameters of outer flow
w conditions on the wall

2. Formulation of the problem

The flow and heat transfer in a compressible turbulent boundary layer on a permeable plate are calculated using a system of continuity, momentum and energy equation.

$$\frac{\partial \rho u}{\partial x}+\frac{\partial\langle\rho v\rangle}{\partial y}=0,$$
$$\rho\frac{d}{dt}=\rho u\frac{\partial}{\partial x}+\langle\rho v\rangle\frac{\partial}{\partial y},$$
$$\rho\frac{du}{dt}=-\frac{dp}{dx}+\frac{\partial}{\partial y}\left(\eta\frac{\partial u}{\partial y}+\rho\tau\right),$$
$$\rho\frac{dh}{dt}=u\frac{dp}{dx}+\frac{\partial}{\partial y}\left(\frac{\eta}{\mathrm{Pr}}\frac{\partial h}{\partial y}+\rho q_t\right)+\eta\left(\frac{\partial u}{\partial y}\right)^2+\rho\tau\frac{\partial u}{\partial y}.$$

In order to determine the turbulent friction $\rho\tau = -\rho\langle u'v'\rangle$ appearing in the momentum equation, we use the three-parameter model of turbulence [3] generalized to a flow with heat transfer in [8], which includes transport equations for turbulent shear stress $\tau = -\langle u'v'\rangle$, energy of turbulence $E = 0.5\sum\langle u_i'^2\rangle$, and parameter $\omega = E/L^2$. The latter has the physical meaning of vorticity of turbulence and includes the transverse integral scale of turbulence L.

$$\rho\frac{dE}{dt}=-(c\rho\sqrt{E}L+c_1\eta)\frac{E}{L^2}+\frac{\partial}{\partial y}\left(D_E\frac{\partial E}{\partial y}\right)+\rho\tau\frac{\partial u}{\partial y}+c_E\rho E\frac{\partial u}{\partial x},$$
$$\rho\frac{d\tau}{dt}=-(c_5\rho\sqrt{E}L+c_6\eta)\frac{\tau}{L^2}+\frac{\partial}{\partial y}\left(D_\tau\frac{\partial \tau}{\partial y}\right)+c_7\rho E\frac{\partial u}{\partial y},$$
$$\rho\frac{d\omega}{dt}=-(2c\rho\sqrt{E}L+1.4c_1\eta f_\omega)\frac{\omega}{L^2}+\frac{\partial}{\partial y}\left(D_\omega\frac{\partial \omega}{\partial y}\right)+\left[\frac{\tau}{E}+2c_4\,\mathrm{sign}\left(\frac{\partial u}{\partial y}\right)\right]\rho\omega\frac{\partial u}{\partial y}+2c_E\rho\omega\frac{\partial u}{\partial x}.$$

$D_\varphi = a_\varphi\rho\sqrt{E}L+\alpha_\varphi\eta(\varphi = E,\tau,\omega)$, $\quad f_\omega = 1-\frac{1}{2c_1}\left(\frac{L}{E}\frac{\partial E}{\partial y}\right)^2$.

Constants: $c = 0.3$; $c_1 = 5\pi/4$; $c_4 = 0.4$; $c_5 = 3c$; $c_6 = 9c_1$; $c_7 = 0.2$; $a_E = a_\omega = 0.06$; $a_\tau = a_E c_5/c$; $\alpha_E = \alpha_\tau = 1$; $\alpha_\omega = 1.4$; $c_E = 0.7$.

This model is supplemented with the equation for turbulent heat flux $q_T = c_p\langle v'T'\rangle$ [9] appearing in the energy equation.

$$\rho\frac{dq_T}{dt}=-(3c\rho\sqrt{E}L+9c_1\eta f(\mathrm{Pr}))\frac{q_T}{L^2}+\frac{\partial}{\partial y}\left(D_q\frac{\partial q_T}{\partial y}\right)+c_{10}\rho E\frac{\partial h}{\partial y}.$$

Here, $f(\mathrm{Pr}) = \frac{1+d}{2}\frac{\sqrt{\mathrm{Pr}}+1/\sqrt{\mathrm{Pr}}}{1+d\sqrt{\mathrm{Pr}}}$, $D_q = a_q\rho\sqrt{E}L+f(\mathrm{Pr})\eta$, $d = 0.25$, $a_q = a_\tau$, $c_{10} = c_7/\mathrm{Pr}_t^o$, $\mathrm{Pr}_t^o = 0.85$.

The longitudinal pressure gradient dp/dx appearing in the momentum equation is calculated in the general case by the distribution of the Mach number along the wall. In the case at hand of constant Mach number $dp/dx = 0$.

The boundary conditions on the wall ($y = 0$) in the case of suction of gas have the following forms:

$$u=0,\ E=\frac{\partial E}{\partial y}=\tau=0,\ j_w=(\rho v)_w,\ T=T_w \text{ or } \left(\lambda\frac{\partial T}{\partial y}\right)_w=q_w \quad \text{at } y=0.$$

Here, j_w is the mass velocity of gas being sucked off, T_w is the wall temperature, and q_w is the heat flux to the wall. The boundary condition $\partial E/\partial y = 0$ enables one to determine the quantity $\omega_w(x)$ which is unknown. In the case of heat-insulated wall under consideration, the wall temperature T_w was determined from the condition q_w=0.

The boundary conditions on the external boundary of the boundary layer ($y = \delta(x)$) in the case of suction have the following forms:

$$u=u_1(x),\ T=T_1(x),\ E=E_1(x),\ \omega=\omega_1(x)\tau=0 \quad \text{at } y=\delta(x).$$

Here, $u_1(x)$ and $T_1(x)$ are functions which describe an outer flow, and the functions $E_1(x)$ and $\omega_1(x)$ describe the degeneracy of turbulence in this flow. The value of $\delta(x)$ is selected from the condition of smooth conjugation of the solution.

In the initial ($x = 0$) cross-section, the boundary layer was preassigned, in which the width of momentum loss θ_0 corresponded to Reynolds number $\mathrm{Re}_\theta = \rho u_1\theta_0/\eta \approx 10$, with a laminar (according to Blasius) velocity profile u/u_1 and temperature $(T-T_1)/(T_1^*-T_1) = 1-(u/u_1)^2$; the profiles of functions $E(y)$, $\tau(y)$ and $\omega(y)$ were preassigned as in [10]. The intensity of turbulence of the flow in the cross-section $x = 0$ was taken to be $e_{10}| = \sqrt{E}_{10}/u_1 = 0.03$.

3. Laminar boundary layer with asymptotic suction

As was mentioned in Section 1, the laminarization of the boundary layer may occur due to suction of gas. In view of this, it appears advisable to analyze the solution for a laminar boundary layer with suction. It is known [6] that, at far distance from the point where suction begins, the velocity and temperature profiles are independent of current length x (asymptotic suction) and are described by the laws of conservation of mass, momentum, and energy (for stagnation enthalpy, $h^o = h+0.5u^2$) in the form

$$\rho v = j_w = const, \tag{1}$$
$$j_w\frac{du}{dy}=\frac{d}{dy}\left(\eta\frac{du}{dy}\right), \tag{2}$$
$$j_w\frac{dh^o}{dy}=\frac{d}{dy}\left\{\frac{\eta}{\mathrm{Pr}}\left[\frac{dh^o}{dy}+(Pr-1)u\frac{du}{dy}\right]\right\}. \tag{3}$$

For a heat-insulated wall, the boundary conditions are on the wall and outer

$$y = 0: \quad u = 0, \; \frac{dh^o}{dy} = 0; \quad y \to \infty: \quad u \to u_1, \; h^0 \to h_1^o. \tag{4}$$

Eq. (2) with account of boundary conditions (4) gives

$$j_w(u - u_1) = \eta \frac{du}{dy}. \tag{5}$$

Integration of Eq. (3) gives

$$j_w(h^o - h_1^o) = \frac{\eta}{\Pr}\frac{du}{dy}\left[\frac{dh^o}{du} + (\Pr - 1)u\right]. \tag{6}$$

At $y = 0$, it follows from Eq. (5) follows $-j_w u_1 = (\eta \partial u/\partial y)_w$ and from Eq. (6) $-j_w(h_w - h_1^o) = q_w = -[(\eta/\Pr)(\partial h/\partial y)]_w$. Therefore, for a heat-insulated wall ($q_w = 0$), the enthalpy of gas on the wall h_w is equal to the stagnation enthalpy in the outer flow h_1^o, and the recovery factor for enthalpy is $r_h = (h_w - h_1)/(h_1^o - h_1) = 1$ irrespectively of Prandtl number.

Let us consider the case where the viscosity of the gas depends on temperature, and the Prandtl number is constant (Pr = *const*).

We transform the relation (6) using (5),

$$\frac{dh^o}{du} + (\Pr - 1)u = -\Pr\frac{h^o - h_1^o}{u_1 - u}.$$

A solution to the resultant first-order linear equation for determining h^o may be readily found. We make the substitution of the variables $H = h^o - h_1^{o\prime}$ and $U = u_1 - u$. For determining H we derive

$$\frac{dH}{dU} + (\Pr - 1)(U - u_1) = \Pr\frac{H}{U}. \tag{7}$$

The solution to Eq. (7) with boundary conditions (4) has the form

$$\begin{aligned} &\frac{h^o}{u_1^2} = \frac{h_1^o}{u_1^2} - \frac{V^{\Pr}}{\Pr - 2} + \frac{\Pr - 1}{\Pr - 2}V^2 - V, \;\; V = \frac{U}{u_1} = 1 - \frac{u}{u_1} \quad (\Pr \neq 2), \\ &\frac{h^o}{u_1^2} = \frac{h_1^o}{u_1^2} + V^2(1 - \ln V) - V \quad (\Pr = 2). \end{aligned} \tag{8}$$

At $y = 0$ ($V = 1$), we will have $h^o = h_1^o$.

Note that the solution (8) at $\Pr \neq 2$ coincides with that obtained using Crocco variables and referred to in [11].

For completing the construction of solution, we must find the dependence of the function V on the coordinate ξ. We will use for this purpose Eq. (5) written in dimensionless form

$$\eta^o\frac{dV}{d\xi} = -V, \quad \eta^o = \frac{\eta}{\eta_1}, \quad \xi = -\frac{j_w y}{\eta_1} \tag{9}$$

with the boundary condition $V = 1$ at $\xi = 0$.

The solution to Eq. (9) has the form

$$\int_1^V \frac{\eta^o}{V}dV = -\xi. \tag{10}$$

The temperature dependence of viscosity of gas must be preassigned to use the formula (10). As an example, we will consider the power law $\eta^o = (T/T_1)^n$.

The temperature is calculated by the relation $h^o = c_pT + 0.5u^2$:

$$\frac{T}{T_1} = 1 + r\frac{\gamma - 1}{2}M_1^2, \quad M_1^2 = \frac{u_1^2}{\gamma R T_1}, \quad \gamma = \frac{c_p}{c_v}.$$

Here, c_p and c_v are the specific heat capacities of the gas at constant pressure and constant volume, respectively.

The current recovery factor for temperature r is determined using formulas (8) and (9),

$$\begin{aligned} &r = V(2 - V) - \frac{2V^{\Pr}}{\Pr - 2} + 2\frac{\Pr - 1}{\Pr - 2}V^2 - 2V \quad (\Pr \neq 2), \\ &r = V(2 - V) + 2V^2(1 - \ln V) - 2V \quad (\Pr = 2). \end{aligned}$$

The indefinite integral in Eq. (10) may be taken for any integer n.

For example, for $n = 1$, we substitute V by the formula $V = 1 - w$ ($w = u/u_1$) to derive

$$\begin{aligned} &\ln(1-w) + \tfrac{\gamma-1}{2M}{}^2_1\left\{\tfrac{2}{\Pr(\Pr-2)}[(1-w)^{\Pr} - 1] + \tfrac{\Pr}{2(\Pr-2)}w(2-w)\right\} = -\xi \;\; (\Pr \neq 2), \\ &\ln(1-w) + (\gamma-1)M_1^2\{(1-w^2)[1 - 2\ln(1-w)] - 1\} = -\xi \;\; (\Pr = 2). \end{aligned}$$

For Pr = 1, the resultant solution coincides with that given in [6].

For constant viscosity $\eta^o = 1$ ($n = 0$), Eq. (5) in view of boundary conditions (4) may be integrated as

$$u = u_1(1 - e^{-\xi}). \tag{11}$$

This solution is given in [6].

After substitution of Eq. (11) into (6), we obtain the solution for stagnation enthalpy h^o. The solution of Eq. (6) is the sum of the general solution of the homogeneous equation and the particular solution of the inhomogeneous equation. The general solution of homogeneous equation (6) is

$$h^o = Ce^{-\Pr\xi}. \tag{12}$$

At Pr = 1 and Pr = 2, finding the general solution calls for special consideration. Let Pr = 1. In this case, the velocity drops out from the equation for determination of h^o, and we derive the following relation for h^o:

$$h^o = h_1^o + Ce^{-\Pr\xi}.$$

It follows from the boundary condition at $y = 0$ that $C = 0$, and the general solution is $h^o = h_1^o$.

Let $\Pr \neq 1$ and $\Pr \neq 2$ (the case of Pr = 2 will be considered further). We derive the following solution from Eq. (6) for h^o:

$$\frac{h^o}{u_1^2} = \frac{h_1^o}{u_1^2} - \frac{1}{\Pr - 2}e^{-\Pr\xi} - e^{-\xi} + \frac{\Pr - 1}{\Pr - 2}e^{-2\xi}. \tag{13}$$

For $y = 0$, we use the latter relation for stagnation enthalpy of gas on the wall to find $h_w = h_1^o$ which was shown above to be independent of Prandtl number.

We consider in conclusion the case of Pr = 2. The solution of Eq. (6) in this case is

$$\frac{h^o}{u_1^2} = \frac{h_1^o}{u_1^2} + \left(1 - \frac{j_w y}{\eta}\right)e^{-2\xi} - e^{-\xi}. \tag{14}$$

We assume $y = 0$ in the latter formula and again obtain $h_w = h_1^o$ on the plate surface.

Assuming the constant specific isobaric heat capacity ($c_p = const$), one can use Eqs. (13) and (14) and derive expressions for temperature profile of the form

$$\begin{aligned} &\Theta = \frac{T^o - T_1}{T_1^o - T_1} = 1 + 2\left(\frac{1}{2 - \Pr}e^{-\Pr\xi} - e^{-\xi} + \frac{1 - \Pr}{2 - \Pr}e^{-2\xi}\right) \;\; (\Pr \neq 2), \\ &\Theta = 1 + 2e^{-\xi}[(1 + \xi)e^{-\xi} - 1] \;\; (\Pr = 2). \end{aligned} \tag{15}$$

4. Integral relations of momentum and energy

For the boundary layer on a permeable wall in a compressible gas flow at constant velocity $u_1 = const$, the integral relations of momentum and energy have the known form

$$\begin{aligned} &\frac{d}{dx}\int_o^\infty \rho u(u_1 - u)dy - j_w u_1 = \tau_w, \\ &\frac{d}{dx}\int_o^\infty \rho u(h^o - h_1^o)dy - j_w(h_w - h_1^o) = q_w. \end{aligned} \tag{16}$$

Here, $j_w = (\rho v)_w$ is the mass velocity of injection (suction) through permeable wall, $\tau_w = (\eta \partial u/\partial y)_w$ is the wall friction, $q_w = (\lambda \partial T/\partial y)_w$ is the heat flux to the wall, h^o is the stagnation enthalpy, and h_w is the enthalpy of gas on the wall.

One can introduce the integral characteristics of a boundary layer

$$\theta = \int_o^\infty \frac{\rho u}{\rho_1 u_1}\left(1 - \frac{u}{u_1}\right) dy,$$
$$\theta_h = \int_o^\infty \frac{\rho u}{\rho_1 u_1}\left(\frac{h^o - h_1}{h_1^o - h_1} - 1\right) dy, \tag{17}$$

and write the relations (16) in the following form:

$$\frac{d}{dx}(\rho_1 u_1^2 \theta) - j_w u_1 = \tau_w, \quad \frac{d}{dx}[\rho_1 u_1 (h_1^o - h_1)\theta_h] - j_w(h_w - h_1) = q_w. \tag{18}$$

Note that integral relations (18) are valid for both laminar and turbulent boundary layers. The difference is in definition of the stagnation enthalpy h^o which, as it follows from the equation of energy conservation, is $h^o = h + u^2/2 + E$ for a turbulent boundary layer, where E is the energy of turbulence.

It may be readily demonstrated that, in an outer flow, where the value of stagnation enthalpy h_1^o is maintained constant, the static (thermodynamic) enthalpy h_1 (temperature T_1) increases owing to the dissipation of energy of turbulence E_1 at constant velocity u_1. In the case of moderate values of Mach number and low value of the initial intensity of turbulence $e_{10} = \sqrt{E_{10}}/u_1$, the rise of temperature T_1 is insignificant. The maximal increase in temperature T_1 (with e_1 decreasing to zero) gives a value $\Delta T = T_{10} e_{10}^2 (\gamma - 1)M^2$; for $M = 3$, $e_{10} = 0.03$, and $T_{10} = 100$ K, this increase is 0.5 K.

If we ignore the variation of the temperature in an outer flow for the turbulent mode of the flow, then we can assume (as for the laminar mode of flow) the enthalpy h_1 (temperature T_1) of the outer flow and, therefore, the density ρ_1 to be constant, i.e., not changing along the flow. Then the integral relations (18) may be written as

$$\frac{d\theta}{dx} = j_w^o + \frac{c_f}{2}, \quad \frac{d\theta_h}{dx} = j_w^o(r_h - 1) + q_w^o. \tag{19}$$

Here, $j_w^o = j_w/\rho_1 u_1$ is the intensity of injection (suction), $c_f = 2\tau_w/\rho_1 u_1^2$ is the friction coefficient, $r_h = (h_w - h_1)/(h_1^o - h_1)$ is the recovery factor for the enthalpy, and $q_w^o = q_w/\rho_1 u_1 (h_1^o - h_1)$ is the dimensionless heat flux to the wall, which equals zero for heat-insulated wall.

We can introduce the Reynolds number for θ and θ_h and for length x, respectively,

$$\mathrm{Re}_\theta = \theta\left(\frac{\rho u}{\eta}\right)_1, \quad \mathrm{Re}_h = \theta_h\left(\frac{\rho u}{\eta}\right)_1, \quad \mathrm{Re}_x = x\left(\frac{\rho u}{\eta}\right)_1,$$

and write the integral relations (19) in the form

$$\frac{d\,\mathrm{Re}_\theta}{d\,\mathrm{Re}_x} = j_w^o + \frac{c_f}{2}, \frac{d\,\mathrm{Re}_h}{d\,\mathrm{Re}_x} = j_w^o(r_T - 1). \tag{20}$$

Here, $q_w^o = 0$ and r_T is the recovery factor for temperature, which is equal to the recovery factor for enthalpy r_h at constant specific heat capacity, $c_p = const.$

For a laminar boundary layer with asymptotic suction of incompressible fluid ($\rho = \rho_1$) at constant viscosity ($\eta = \eta_1$) and heat capacity ($c_p = const$), one can use solutions for velocity profile (11) and temperature (15) and derive the following dimensionless dependences for the boundary layer integral characteristics (17):

$$\mathrm{Re}_\theta j_w^o = 1/2. \tag{21}$$

$$\mathrm{Re}_h j_w^o = \frac{6 - 5\mathrm{Pr} - 3\mathrm{Pr}^2 + 2\mathrm{Pr}^3}{3\mathrm{Pr}(1+\mathrm{Pr})(2-\mathrm{Pr})}\ (\mathrm{Pr} \neq 2), \quad \mathrm{Re}_h j_w^o = -\frac{7}{18}\ (\mathrm{Pr} = 2). \tag{22}$$

5. Calculation results

The calculations were performed in the following formulation (Fig. 1). A plate was subjected to a gas flow at longitudinally constant supersonic velocity u_1 at temperature $T_1 = 100$ K. The stagnation temperature T_1^o depends on the Mach number of an outer flow M, which is a parameter of the problem, as well as the Prandtl number Pr and Reynolds number with respect to length $\mathrm{Re}_x = x(\rho u/\eta)_1$, for values of the thermal properties determined by the temperature of outer flow. The outer flow Mach number in calculations was taken to be $M = 3$; in so doing, the stagnation temperature was $T_1^o = 400$ K, and the Reynolds number beginning from which the intensity of suction was constant along the permeable plate was $\mathrm{Re}_x = 10^6$.

The plate region of length x_0 (Fig. 1) was assumed to be impermeable and heat-insulated ($q_w = 0$). Further downstream, the gas suction was performed, the intensity of which $j_w^o = j_w/(\rho u)_1$ was linearly increasing over a short length Δx and then stayed constant along the plate. The inlet region length x_0 in calculations was selected to be such that the beginning of suction was located behind the region of transition from the laminar to turbulent mode of flow in the boundary layer.

The gas in calculations was helium, the thermal properties of which [12] are close to those of ideal gas: the specific isobaric heat capacity is $c_p = const$, and the dynamic viscosity η and thermal conductivity λ depend on temperature alone so that the Prandtl number is $\mathrm{Pr} = \eta c_p/\lambda = const$.

The variation of the integral and local characteristics of the boundary layer with respect to length (Reynolds number Re_x) was investigated for the aim to determine the extent of the region in which the asymptotic solution of the equation for boundary layer with suction is derived.

The investigation was performed for two values of Prandtl number Pr = 0.1 and 4.0 and for three values of intensity of suction $j_w^o = -0.005, -0.01, -0.02$. Fig. 2 gives the variation of $2j_w^o/c_f$ and Fig. 3 – the variation of the recovery factor for temperature r_T.

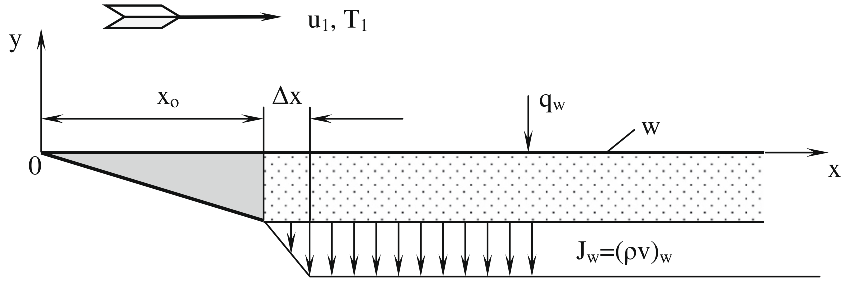

Fig. 1. Calculation scheme.

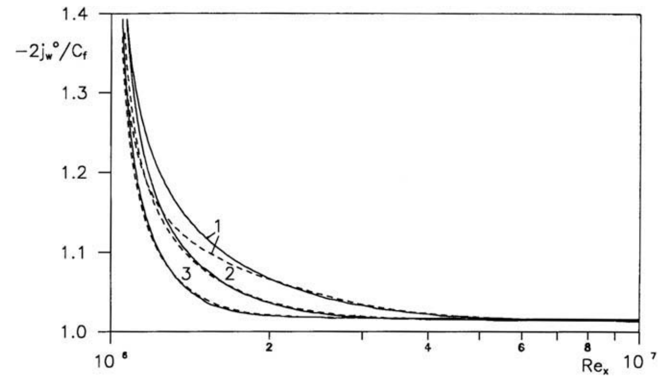

Fig. 2. Variation of the ratio between the suction intensity j_w^o and the friction coefficient $c_f/2$ along the permeable plate for a number of values of j_w^o: line 1, $j_w^o = -0.005$; line 2, $j_w^o = -0.01$; line 3, $j_w^o = -0.02$; continuous lines, Prandtl number Pr = 0.1; dashed lines, Pr = 4.

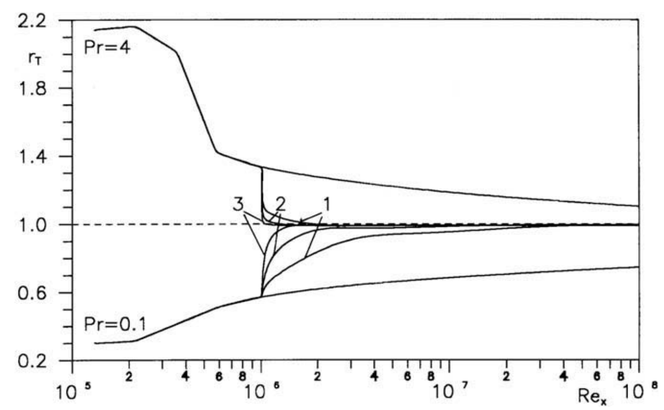

Fig. 3. Variation of the recovery factor for temperature r_T along the permeable plate for a number of values of suction intensity j_w^o: line 1, $j_w^o = -0.005$; line 2, $j_w^o = -0.01$; line 3, $j_w^o = -0.02$, for the values of Prandtl number Pr = 0.1 and 4.

One can see that, starting with a certain distance from the beginning of suction ($Re_x > 10^6$), the friction coefficient $c_f/2$ (Fig. 2) becomes close (within less than 1.5%) to the intensity of suction $-j_w^o$, and the recovery factor becomes (within less than 1%) $r_T = 1$ (Fig. 3). Its means that, according to Eq. (20), the numbers Re_θ and Re_h at $Re_x < 10^7$ become constant and do so faster as the suction intensity increases.

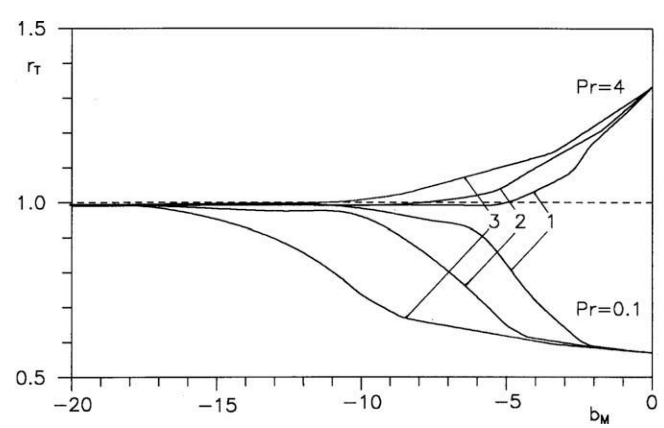

Fig. 4. The recovery factor for temperature r_T as a function of permeability parameter b_M for a number of values of suction intensity j_w^o (designations are the same as in Fig. 3).

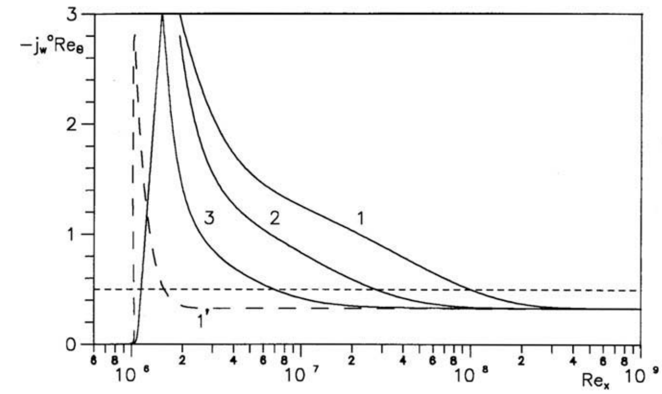

Fig. 5. Variation of the product of Reynolds number Re_θ by suction intensity j_w^o (for the value of Prandtl number Pr = 0.1) along the permeable plate for a number of values of suction intensity j_w^o: line 1, $j_w^o = -0.005$; line 2, $j_w^o = -0.01$; line 3, $j_w^o = -0.02$; line 1′, laminar boundary layer at $j_w^o = -0.005$.

Fig. 4 gives the recovery factor for temperature r_T as a function of permeability parameter $b_m = j_w^o/St_M$ (St_M is the Stanton number at $M = 3$), which illustrates the rate at which the asymptotic solution is reached with respect to parameter b_m.

It follows from Eqs. (21) and (22) in the case of constant viscosity that the products of Reynolds numbers Re_θ and Re_h by the intensity of suction j_w^o are independent of the value of j_w^o and are constant for asymptotic boundary layer, $Re_\theta j_w^o = -1/2$, and $Re_h j_w^o$ depends on Prandtl number alone. Figs. 5 and 6 give the variation of $Re_\theta j_w^o$ as a function of Re_x. One can see that $Re_\theta j_w^o$ indeed assumes a constant value (other than $-1/2$) and does so the faster, the higher the intensity of suction, but much slower than in the case of initial laminar boundary layer (lines 1′ in Figs. 5 and 6).

The quantity $Re_h j_w^o$ (Figs. 7 and 8) is slower than $Re_\theta j_w^o$ in assuming a constant value (especially, for a low value of Prandtl number Pr) and differs from the quantities which correspond to constant viscosity (22), i.e., $Re_h j_w^o = -8.727$ for Pr = 0.1 and $Re_h j_w^o = -0.55$ for Pr = 4.

The investigation of the behavior of local characteristics in the case of suction revealed the following. The velocity profiles in coordinates u/u_1, $\xi = -y j_w/\eta_1$ (Fig. 9) are independent of the intensity of suction ($-j_w^o = 0.005 - 0.02$) and of Reynolds number ($Re_x > 10^7$); they differ only by the Prandtl number Pr (lines 1 and 2 in Fig. 9) and agree with the results of calculations for the initial laminar boundary layer, which is indicative of asymptotic solution for the parameters identified above. The results further differ from the solution for incompressible fluid of constant

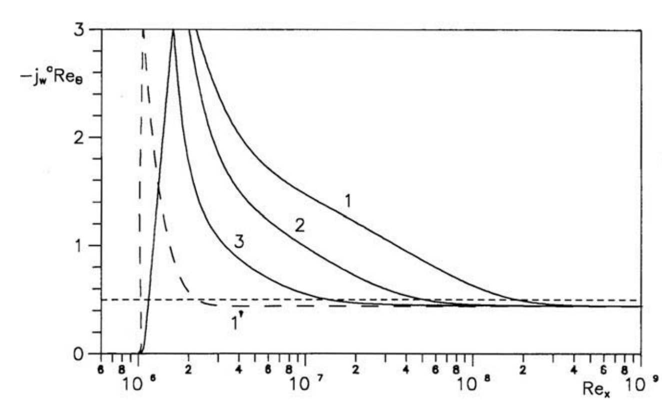

Fig. 6. See Fig. 5 (for the value of Prandtl number Pr = 4).

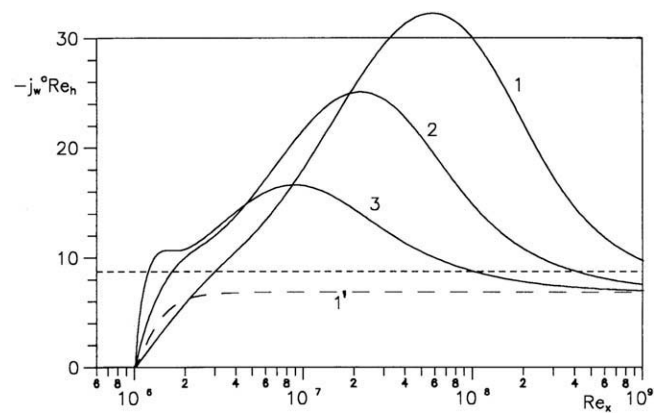

Fig. 7. Variation of the product of Reynolds number Re_h by suction intensity j_w^o (for the value of Prandtl number Pr = 0.1) along the permeable plate for a number of values of j_w^o (designations are the same as in Fig. 5).

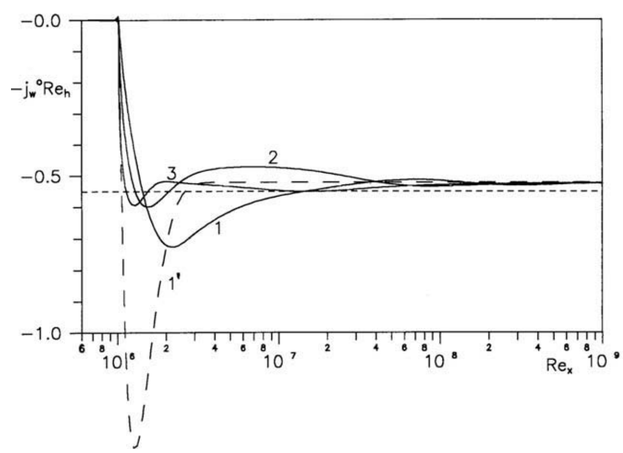

Fig. 8. See Fig. 7 (for the value of Prandtl number Pr = 4).

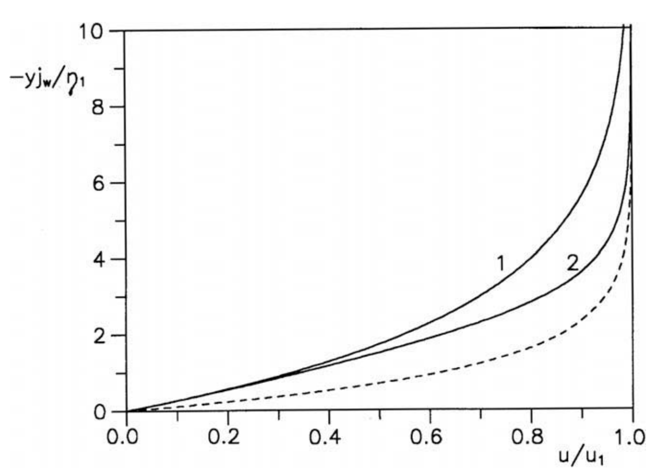

Fig. 9. Profiles of velocity u/u_1 in boundary layer with suction for the value of Reynolds number $Re_x = 10^8$: line 1, Pr = 0.1; line 2, Pr = 4; dashed lines indicate solution (11) for incompressible fluid of constant viscosity at the values of Prandtl number Pr = 0.1 and 4.0.

viscosity (11) (dashed line in Fig. 9). Similar results were obtained for temperature profiles in coordinates $\Theta = (T^o - T_1)/(T_1^o - T_1)$, $\xi = -yj_w/\eta_1$ (Fig. 10).

The effect of suction on the profiles of velocity u/u_1, temperature T/T_1, and intensity of turbulence $e = \sqrt{E}/u_1$ over the boundary layer thickness y/δ is illustrated in Figs. 11–13. One can see that a significant deformation of the profiles of velocity (Fig. 11) and temperature (Fig. 12) occurs in the case of suction, and the degeneracy of energy of turbulence (Fig. 13) is directly indicative of laminarization of boundary layer under conditions of suction, which is the necessary condition of deriving the asymptotic solution.

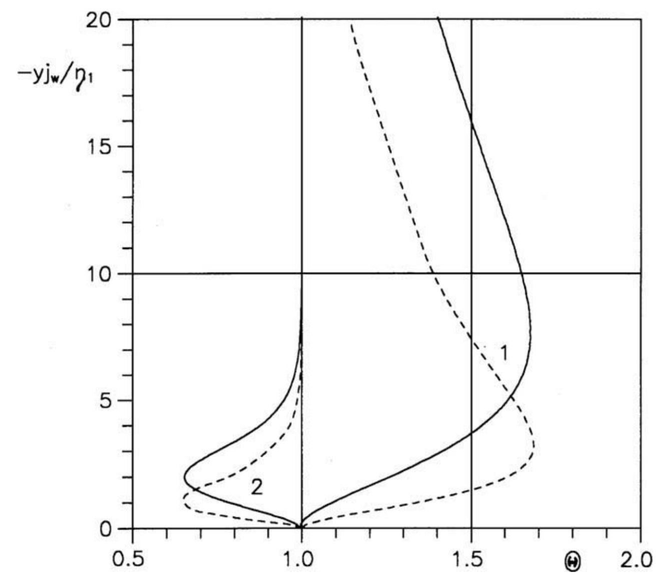

Fig. 10. Profiles of temperature $\Theta = (T^o - T_1)/(T_1^o - T_1)$ in boundary layer with suction for the value of Reynolds number $Re_x = 10^8$: line 1, Pr = 0.1; line 2, Pr = 4; dashed lines indicate solution (15) for incompressible fluid of constant viscosity at the values of Prandtl number Pr = 0.1 and 4.0.

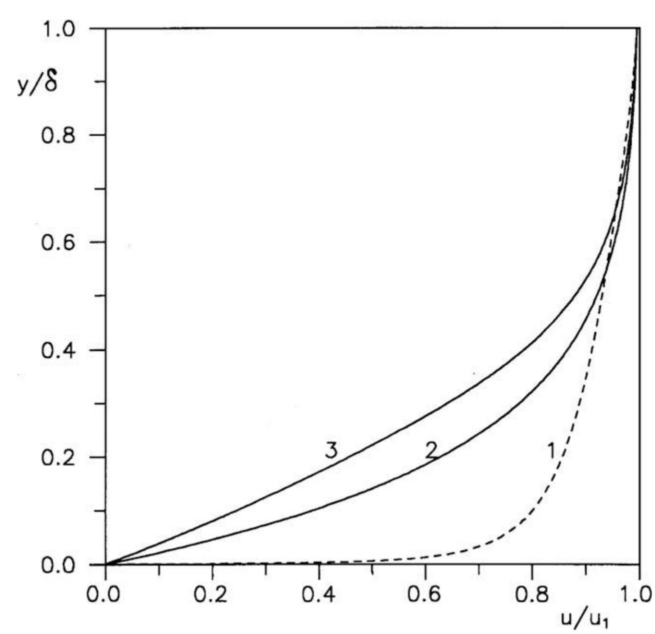

Fig. 11. The effect of suction on the profiles of velocity u/u_1 over the boundary layer thickness y/δ for the value of Reynolds number $Re_x = 10^8$: line 1, Pr = 0.1 and 4; lines 2 and 3, $j_w^o = -0.01$ at the values of Prandtl number Pr = 0.1 and 4, respectively.

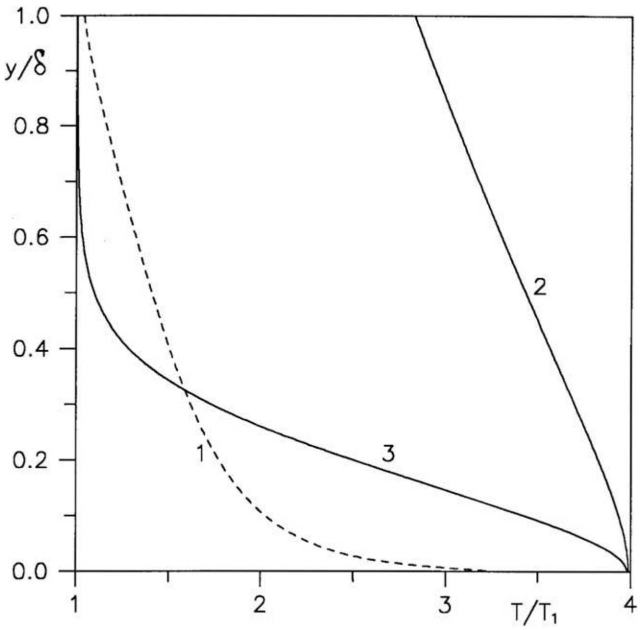

Fig. 12. The effect of suction on the profiles of temperature T/T_1 over the boundary layer thickness y/δ (designations are as in Fig. 11).

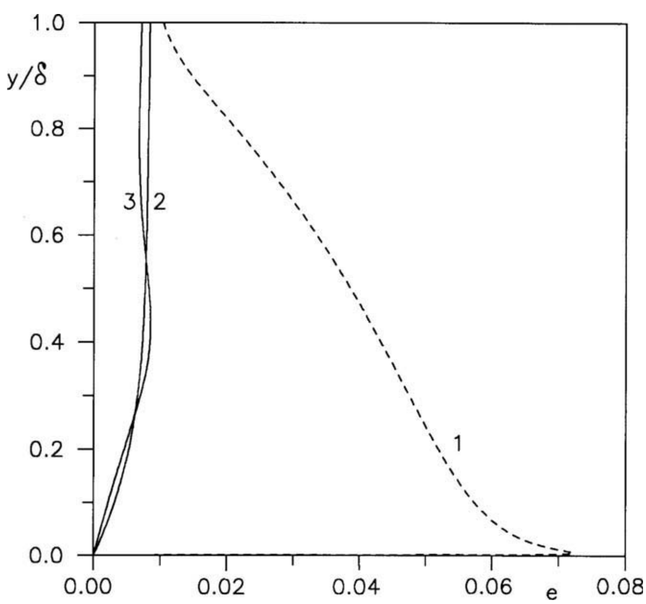

Fig. 13. The effect of suction on the profiles of intensity of turbulence $e = \sqrt{E}/u_1$ over the boundary layer thickness y/δ (designations are as in Fig. 11).

6. Conclusion

The numerical investigation of the boundary layer on a heat-insulated wall with suction in a compressible gas flow involving the use of differential model of turbulence supplemented with transport equation for turbulent heat flux revealed the following:

1. The laminarization of the initial turbulent boundary layer occurs under conditions of suction of gas, as is evidenced both by the behavior of integral and local characteristics of flow and heat transfer and by the degeneracy of turbulence when the suction becomes asymptotic; the asymptotic suction is characterized by constant value of mass transverse velocity and by developed profiles of temperature and longitudinal velocity.
2. The temperature recovery factor in the case of asymptotic suction is independent of Prandtl number and becomes equal to unity, i.e., the temperature of the heat-insulated wall becomes equal to the stagnation temperature of outer flow.

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